

### 3 Methods of Proof

**17.** Show that if  $n$  is an integer and  $3n + 2$  is an odd number, then  $n$  is an odd number.

**18.** Prove by contradiction that the number  $\log_{\sqrt{2}} 3$  is irrational.

**19.** Show that if  $n$  is an even number, then  $n^2 + 3n$  is even.

Is the converse true?

**20.** Let  $n$  be an integer ( $n \in \mathbb{Z}$ ). Prove that  $n^2 \geq n$ .

**21.** Let  $x$  and  $y$  be real numbers such that  $x < 2y$ . Prove the following implication:

$$\text{If } 7xy \leq 3x^2 + 2y^2, \text{ then } 3x \leq y.$$

**22.** Show that if  $n$  is an odd integer and  $m$  is an even integer, then  $n^2 + 3n + nm + 1$  is an odd number. Explain each step of your proof!

**23. (exam, November 2021)** Show that if  $3nm + n + m$  is an even number, then both  $n$  and  $m$  are even numbers.

**24.** Consider the statement:

If  $x$  and  $y$  are odd integers, then their product  $xy$  is also an odd integer.

(i) Provide a direct proof of the above implication.

(ii) Provide a proof by contradiction of the above implication.

**25.** The reciprocal of a positive real number  $x$  is  $\frac{1}{x}$ . Consider the statement:

- If a real number  $x$  is positive, then the sum of  $x$  and its reciprocal is greater than or equal to 2.

(i) Provide a direct proof of the above implication.

(ii) Provide a proof by contradiction of the above implication.

All above math problems are taken from the following website:

<https://osebje.famnit.upr.si/~penjic/teaching.html>.

THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.